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# Signals of $CP$ violation in Higgs decay

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## Abstract

We consider an extension of the Standard Model where some Higgs particle is not an eigenstate of  $CP$ , and discuss the possibility of extracting signals of the resulting  $CP$  violation. In the case of Higgs decay to four fermions we study correlations among momenta of the final-state fermions. We discuss observables which may demonstrate presence of  $CP$  violation and identify a phase shift  $\delta$ , which is a measure of the strength of  $CP$  violation in the Higgs-vector-vector coupling, and which can be measured directly in the decay distribution. In addition to these angular correlations, we consider correlations between energy differences. The former correlations include some recently reported results, whereas the latter ones appear to provide a much better probe for revealing  $CP$  violation.

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The origin of mass and the origin of  $CP$  violation are widely considered to be the most fundamental issues in contemporary particle physics. Perhaps for this very reason, there has been much speculation on a possible connection. This is also in part caused by the fact that the amount of  $CP$  violation accommodated in the CKM matrix does not appear sufficient to explain the observed baryon to photon ratio [1].

We shall here consider the possibility that  $CP$  violation is present in the Higgs sector, and discuss some possible signals of such effects in decays of Higgs particles. While the standard model induces  $CP$  violation in the Higgs sector at the one-loop level provided the Yukawa couplings contain both scalar and pseudoscalar components [2], we actually have in mind an extended model, such as e.g., the two-Higgs-doublet model [3].

Below we postulate an effective Lagrangian which contains  $CP$  violation in the Higgs sector. However, when the source of  $CP$  violation is turned off, the interaction reduces to that of the Standard Model. In cases considered in the literature,  $CP$  violation usually appears as a one-loop effect. This is due to the fact that the  $CP$  odd coupling introduced below is a higher-dimensional operator and in renormalizable models these are induced only at loop level. Consequently we expect the effects to be small and any observation of  $CP$  violation to be equally difficult.

$CP$  non-conservation has manifested itself so far only in the neutral kaon system. In the context of the Standard Model this  $CP$  violation originates from the Yukawa sector via the CKM matrix [4]. Although there may be several sources of  $CP$  violation, including the one above, we will here consider a simple model where the  $CP$  violation is restricted to the Higgs sector and in particular to the coupling between some Higgs boson and the vector bosons. Specifically, by assuming that the coupling between the Higgs boson  $H$  and the vector bosons  $V = W, Z$  has both scalar and pseudoscalar components, the effective Lagrangian for the  $HVV$ -vertex may be written as [5, 6, 7]

$$\mathcal{L}_{HVV} = 2 \cdot 2^{1/4} \sqrt{G_F} \left[ m_V^2 V^\mu V_\mu H + \frac{1}{4} \eta \epsilon^{\mu\nu\rho\sigma} V_{\mu\nu} V_{\rho\sigma} H \right], \quad (1)$$

with  $G_F$  the Fermi constant. Furthermore,  $V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$ . The parameter  $\eta$  is, according to the discussion above, a small dimensionless quantity that depends on the kinematics. However, we will keep  $\eta$  arbitrary and our results are valid for any value of

$\eta$ . The first term in  $\mathcal{L}_{HVV}$  is  $CP$  even, whereas the second one is  $CP$  odd. Simultaneous presence of both terms leads to  $CP$  violation. However, we assume that  $CPT$  is conserved. This implies that  $\eta$  must be real.

We shall consider the decay of a Higgs via two vector bosons ( $W^+W^-$  or  $ZZ$ ), to two non-identical fermion-antifermion pairs,  $H \rightarrow V_1V_2 \rightarrow (f_1\bar{f}_2)(f_3\bar{f}_4)$ . Let the momenta of the two fermion-antifermion pairs ( $q_1, q_2, q_3$ , and  $q_4$  in the Higgs rest frame) define two planes, and denote by  $\phi$  the angle between those two planes (see eq. (9) below). Then we shall discuss the angular distribution of the decay rate  $\Gamma$ ,

$$\frac{1}{\Gamma} \frac{d\Gamma}{d\phi} \quad (2)$$

and an energy-weighted decay rate, to be defined below.

Related studies have been reported by [5, 7]. The present discussion will be more general and in addition we consider correlations between energy differences. Under suitable experimental conditions these energy correlations provide a better signal of  $CP$  violation.

The fermion-vector coupling is given by

$$-\frac{i}{2\sqrt{2}}\gamma^\mu(g_V - g_A\gamma_5),$$

where  $g_V$  and  $g_A$  denote the vector and axial-vector parts of the couplings. As a parameterization of these, we define the angles  $\chi_1$  and  $\chi_2$  by

$$g_V^{(i)} \equiv g_i \cos \chi_i, \quad g_A^{(i)} \equiv g_i \sin \chi_i, \quad i = 1, 2. \quad (3)$$

The only reference to these angles is through  $\sin 2\chi_i$ . Relevant values are given in table 1 of ref. [8]. From (1), the coupling of  $H$  to the vector bosons is given by

$$i \left( 2 \cdot 2^{1/4} \right) \sqrt{G_F} \left[ m_V^2 g^{\mu\nu} + \eta \left( k_1^2, k_2^2 \right) \epsilon^{\mu\nu\rho\sigma} k_{1\rho} k_{2\sigma} \right], \quad (4)$$

with  $k_j$  the momentum of vector boson  $j$ ,  $j = 1, 2$ .

The decay rate can be written as

$$d^8\Gamma = \sqrt{2} \frac{G_F}{m} N_1 N_2 D \left[ X + \frac{\eta}{m_V^2} Y + \left( \frac{\eta}{m_V^2} \right)^2 Z \right] d\text{Lips}(m^2; q_1, q_2, q_3, q_4), \quad (5)$$

with  $m$  the Higgs mass and  $\text{dLips}(m^2; q_1, q_2, q_3, q_4)$  denoting the Lorentz-invariant phase space. Furthermore,  $N_j$  is a colour factor, which is three for quarks, and one for leptons. The momentum correlations are in the massless fermion approximation given by

$$\begin{aligned} X &= X_+ + \sin 2\chi_1 \sin 2\chi_2 X_-, \\ Y &= -\epsilon_{\alpha\beta\gamma\delta} q_1^\alpha q_2^\beta q_3^\gamma q_4^\delta [Y_- + \sin(2\chi_1) \sin(2\chi_2) Y_+], \\ Z &= -2X_-^2 + \frac{1}{4} s_1 s_2 [Z_1 + \sin(2\chi_1) \sin(2\chi_2) Z_2], \end{aligned} \quad (6)$$

where

$$\begin{aligned} X_\pm &= (q_1 \cdot q_3)(q_2 \cdot q_4) \pm (q_1 \cdot q_4)(q_2 \cdot q_3), \\ Y_\mp &= (q_1 \mp q_2) \cdot (q_3 \mp q_4), \\ Z_1 &= [(q_1 \cdot q_3) + (q_2 \cdot q_4)]^2 + [(q_1 \cdot q_4) + (q_2 \cdot q_3)]^2 - \frac{1}{2} s_1 s_2, \\ Z_2 &= [(q_1 + q_2) \cdot (q_3 - q_4)][(q_1 - q_2) \cdot (q_3 + q_4)]. \end{aligned} \quad (7)$$

The normalization in eq. (5) involves the function

$$D(s_1, s_2) = m_V^4 \prod_{j=1}^2 \frac{g_j^2}{(s_j - m_V^2)^2 + m_V^2 \Gamma_V^2}, \quad (8)$$

with

$$s_1 \equiv (q_1 + q_2)^2, \quad s_2 \equiv (q_3 + q_4)^2.$$

Finally,  $m_V$  and  $\Gamma_V$  denote the mass and total width of the relevant vector boson, respectively.

We first consider angular correlations. The relative orientation of the two planes is defined by the angle  $\phi$ ,

$$\cos \phi = \frac{(\mathbf{q}_1 \times \mathbf{q}_2) \cdot (\mathbf{q}_3 \times \mathbf{q}_4)}{|\mathbf{q}_1 \times \mathbf{q}_2| |\mathbf{q}_3 \times \mathbf{q}_4|}. \quad (9)$$

We find

$$\begin{aligned} \frac{d^3\Gamma}{d\phi ds_1 ds_2} &= \frac{\sqrt{2}}{72(4\pi)^6} N_1 N_2 \frac{G_F}{m^3} \sqrt{\lambda(m^2, s_1, s_2)} D(s_1, s_2) \\ &\times \left[ X' + \frac{\eta}{m_V^2} \sqrt{\lambda(m^2, s_1, s_2)} Y' + \left( \frac{\eta}{m_V^2} \right)^2 \lambda(m^2, s_1, s_2) Z' \right], \end{aligned} \quad (10)$$

with

$$\begin{aligned}
X' &= \lambda(m^2, s_1, s_2) + 12s_1s_2 + 2s_1s_2 \cos 2\phi \\
&\quad - \sin 2\chi_1 \sin 2\chi_2 \left(\frac{3\pi}{4}\right)^2 \sqrt{s_1s_2} (m^2 - s_1 - s_2) \cos \phi, \\
Y' &= 2s_1s_2 \sin 2\phi - \sin 2\chi_1 \sin 2\chi_2 \frac{1}{2} \left(\frac{3\pi}{4}\right)^2 \sqrt{s_1s_2} (m^2 - s_1 - s_2) \sin \phi, \\
Z' &= 2s_1s_2 \left(1 - \frac{1}{4} \cos 2\phi\right), \tag{11}
\end{aligned}$$

and where  $\lambda(x, y, z) \equiv x^2 + y^2 + z^2 - 2(xy + xz + yz)$  is the usual two-body phase space function.

The distribution (10) can be written in a more compact form as

$$\begin{aligned}
\frac{d^3\Gamma}{d\phi ds_1 ds_2} &= \frac{\sqrt{2}}{72(4\pi)^6} N_1 N_2 \frac{G_F}{m^3} \sqrt{\lambda(m^2, s_1, s_2)} D(s_1, s_2) \\
&\quad \times \left[ \lambda(m^2, s_1, s_2) + 4s_1s_2 (1 + 2\rho^2) + 2s_1s_2 \rho^2 \cos 2(\phi - \delta) \right. \\
&\quad \left. - \sin 2\chi_1 \sin 2\chi_2 \left(\frac{3\pi}{4}\right)^2 \sqrt{s_1s_2} (m^2 - s_1 - s_2) \rho \cos(\phi - \delta) \right], \tag{12}
\end{aligned}$$

with a modulation function

$$\rho = \sqrt{1 + \eta^2 \lambda(m^2, s_1, s_2) / (4m_V^4)}, \tag{13}$$

and an angle

$$\delta = \arctan \frac{\eta(s_1, s_2) \sqrt{\lambda(m^2, s_1, s_2)}}{2m_V^2}, \quad -\pi/2 < \delta < \pi/2, \tag{14}$$

describing the relative shift in the spatial distribution of the two decay planes due to  $CP$  violation. We note that this rotation vanishes at the threshold for producing vector bosons (where  $\lambda = 0$ ) and grows with increasing values of the Higgs mass.

This relation (14) can be inverted to give for the  $CP$ -odd term in the coupling:

$$\eta = \frac{2m_V^2}{\sqrt{\lambda(m^2, s_1, s_2)}} \tan \delta. \tag{15}$$

A more inclusive distribution is obtained if we integrate over the invariant masses of the two pairs. Thus, let us consider

$$\frac{d\Gamma}{d\phi} = \int_0^{m^2} ds_1 \int_0^{(m-\sqrt{s_1})^2} ds_2 \frac{d^3\Gamma}{d\phi ds_1 ds_2}. \tag{16}$$

Due to our ignorance concerning  $\eta = \eta(s_1, s_2)$ , we have to perform the integration over  $s_1$  and  $s_2$  in the narrow-width approximation. This is of course only meaningful above threshold for producing real vector bosons. We introduce the ratio  $\mu = (2m_V/m)^2 < 1$ . The distribution of eq. (2) then takes the compact form

$$\frac{2\pi}{\Gamma} \frac{d\Gamma}{d\phi} = 1 + \alpha(m) \rho^2 \cos 2(\phi - \delta) + \beta(m) \rho \cos(\phi - \delta), \quad (17)$$

with

$$\begin{aligned} \alpha(m) &= \frac{1}{2} \frac{\mu^2}{4(1-\mu)(1+2\eta^2) + 3\mu^2}, \\ \beta(m) &= -\frac{1}{2} \sin 2\chi_1 \sin 2\chi_2 \left(\frac{3\pi}{4}\right)^2 \frac{\mu(2-\mu)}{4(1-\mu)(1+2\eta^2) + 3\mu^2}, \end{aligned} \quad (18)$$

and

$$\delta = \arctan \frac{2\eta\sqrt{1-\mu}}{\mu}, \quad \rho = \sqrt{1 + 4\eta^2 \frac{1-\mu}{\mu^2}}. \quad (19)$$

Measurement of this rotation  $\delta$  of the azimuthal distributions, would demonstrate  $CP$  violation in the coupling between the Higgs boson and the vector bosons. In order to facilitate such a possibility, we introduce the following measures of the asymmetry:

$$\left. \begin{matrix} A(m) \\ A'(m) \end{matrix} \right\} \equiv \frac{1}{\pi} \int_0^{2\pi} d\phi \left\{ \begin{matrix} \cos 2\phi \\ \sin 2\phi \end{matrix} \left( \frac{2\pi}{\Gamma} \frac{d\Gamma}{d\phi} \right) = \rho^2 \alpha(m) \left\{ \begin{matrix} \cos 2\delta \\ \sin 2\delta \end{matrix} \right. \right\}, \quad (20)$$

$$\left. \begin{matrix} B(m) \\ B'(m) \end{matrix} \right\} \equiv \frac{1}{\pi} \int_0^{2\pi} d\phi \left\{ \begin{matrix} \cos \phi \\ \sin \phi \end{matrix} \left( \frac{2\pi}{\Gamma} \frac{d\Gamma}{d\phi} \right) = \rho \beta(m) \left\{ \begin{matrix} \cos \delta \\ \sin \delta \end{matrix} \right. \right\}, \quad (21)$$

where the unprimed observables correspond to the SM prediction (modulo corrections of order  $\eta^2$ ) and the primed ones correspond to the  $CP$  violating contributions. This identification is evident when we note that (see eqs. (10) and (11))

$$\frac{2\pi}{\Gamma} \frac{d\Gamma}{d\phi} = 1 + A(m) \cos 2\phi + B(m) \cos \phi + A'(m) \sin 2\phi + B'(m) \sin \phi \quad (22)$$

for any  $\eta$ . The introduction of the above asymmetries requires that we are experimentally in a position where we can orient  $\phi$  from 0 to  $2\pi$ . (Of course, complete jet identification is required for this kind of analysis. We hope to present Monte Carlo data on the expected efficiency elsewhere.)

Presence of  $CP$  violation will now manifest itself as a non-zero value for the observables  $A'$  and  $B'$ ; the magnitude of  $CP$  violation may be determined from the angle  $\delta$ ,

$$\tan 2\delta = \frac{A'(m)}{A(m)} \quad \text{and} \quad \tan \delta = \frac{B'(m)}{B(m)}. \quad (23)$$

In principle, this allows for consistency checks, since the two expressions in eq. (23) are closely related. However, a possible experimental observation of  $CP$  violation is strongly dependent upon the magnitudes of the observables  $A'$  and  $B'$ . A first question in this direction is: which of them is easier to detect? In order to answer this question, we may study the ratio

$$\frac{B'(m)}{A'(m)} = \frac{\beta(m)}{2\alpha(m)} \quad (24)$$

(we have here made use of the fact that  $\rho \cos \delta = 1$ ). This ratio is *independent of  $\eta$* . In the case of  $H \rightarrow W^+W^- \rightarrow 4f$ ,  $B'$  is relatively important for any Higgs mass. In the case of  $H \rightarrow ZZ \rightarrow 4l$ ,  $A'$  is relatively important for intermediate Higgs bosons, whereas  $B'$  becomes more important for  $m \gtrsim 600$  GeV. (The case e.g.  $H \rightarrow ZZ \rightarrow 2l2q$  is intermediate, cf. table 1 in [8].)

The amplitude functions  $\alpha(m)\rho^2$  and  $\beta(m)\rho$  of (17) are given in figs. 1–2 for the cases  $H \rightarrow W^+W^- \rightarrow 4f$  and  $H \rightarrow ZZ \rightarrow 4l$ , respectively. We note that  $\alpha(m)\rho^2$  is independent of the relative strengths of axial and vector couplings, whereas  $\beta(m)\rho$  is proportional to the  $\sin 2\chi_i$  factors. Both amplitudes are small in most of the  $(\eta, m, \sin 2\chi_i)$  parameter-space, but  $\beta(m)\rho$  is comparable to unity for any value of  $\eta$ , in the intermediate Higgs mass range in the case  $H \rightarrow W^+W^- \rightarrow 4f$ . The two amplitudes are *independent* of the Higgs mass *iff*  $\eta = \pm 1$ .

The angle  $\delta$  is given in fig. 3 for different values of  $\eta$ . Provided  $\eta$  is not too small, this angle is significantly different from zero when the Higgs is well above threshold. It appears from the previous figures that an experimental observation of  $CP$  violation would only be possible in restricted ranges of  $\eta$  and Higgs mass values, and only for selected decay channels.

Let us now turn to a discussion of the decay rate weighted with energy differences. We multiply (5) by the energy differences  $(\omega_1 - \omega_2)(\omega_3 - \omega_4)$  before integrating over energies.

In analogy with eq. (5), we introduce

$$d^8\tilde{\Gamma} = d^8\Gamma(\omega_1 - \omega_2)(\omega_3 - \omega_4), \quad (25)$$

and integrate analytically over kinematic variables to obtain the distribution

$$\begin{aligned} \frac{d^3\tilde{\Gamma}}{d\phi ds_1 ds_2} &= \frac{\sqrt{2}}{288(4\pi)^6} N_1 N_2 \frac{G_F}{m^5} \lambda^{3/2} (m^2, s_1, s_2) D(s_1, s_2) \\ &\times \left[ 2s_1 s_2 \sin 2\chi_1 \sin 2\chi_2 \rho^2 - \left( \frac{3\pi}{16} \right)^2 \sqrt{s_1 s_2} (m^2 - s_1 - s_2) \rho \cos(\phi - \delta) \right]. \end{aligned} \quad (26)$$

In the narrow-width approximation we obtain

$$\frac{2\pi}{\tilde{\Gamma}} \frac{d\tilde{\Gamma}}{d\phi} = 1 + \frac{\kappa(m)}{\rho} \cos(\phi - \delta), \quad (27)$$

with

$$\kappa(m) = -\frac{1}{\sin 2\chi_1 \sin 2\chi_2} \left( \frac{3\pi}{16} \right)^2 \frac{2 - \mu}{\mu} = \frac{1}{(4 \sin 2\chi_1 \sin 2\chi_2)^2} \frac{\beta(m)}{\alpha(m)}, \quad (28)$$

which is independent of  $\eta$ . As in eqs. (20)–(21) we introduce the following measures of asymmetry:

$$\left\{ \begin{array}{l} C(m) \\ C'(m) \end{array} \right\} \equiv \frac{1}{\pi} \int_0^{2\pi} d\phi \left\{ \begin{array}{l} \cos \phi \\ \sin \phi \end{array} \right\} \left( \frac{2\pi}{\tilde{\Gamma}} \frac{d\tilde{\Gamma}}{d\phi} \right) = \left( \frac{\kappa(m)}{\rho} \right) \left\{ \begin{array}{l} \cos \delta \\ \sin \delta \end{array} \right\}, \quad (29)$$

so that

$$\frac{2\pi}{\tilde{\Gamma}} \frac{d\tilde{\Gamma}}{d\phi} = 1 + C(m) \cos \phi + C'(m) \sin \phi. \quad (30)$$

Consequently

$$\tan \delta = \frac{C'(m)}{C(m)}. \quad (31)$$

Hence, this set of observables provides yet another way of measuring the  $CP$  violating phase  $\delta$ . Although the ratio of these energy-weighted observables coincides with, or is trivially related to the previous ones, the possibility of demonstrating a presence of  $CP$  violation has increased significantly. The ratio

$$\frac{C'(m)}{B'(m)} = \frac{1}{(4 \sin 2\chi_1 \sin 2\chi_2)^2 \alpha(m) \rho^2} \quad (32)$$

is given in fig. 4 for  $\eta = 10^{-3}, 10^{-1}$ , and 1 in the cases of  $H \rightarrow W^+W^- \rightarrow 4f$  and  $H \rightarrow ZZ \rightarrow 4l$ . We see that the energy-weighted  $CP$ -violating observable  $C'$  is a much



more sensitive probe for establishing  $CP$  violation than the former ones. Moreover, the amplitude function  $\kappa(m)/\rho$  of (27) is given in fig. 5 for the cases  $H \rightarrow W^+W^- \rightarrow 4f$  and  $H \rightarrow ZZ \rightarrow 4l$ . We observe that this amplitude is comparable to, or much bigger than unity for arbitrary values of  $\eta$  and for Higgs decay to any observable four-fermion final state. In addition, for  $V = Z \rightarrow 2l$  the  $\sin 2\chi$ -factors provide an enhancement in the energy-weighted amplitude function, and particularly in the ratio  $C'/B'$ . It is encouraging that this enhancement occurs for the so-called “gold-plated mode”  $H \rightarrow ZZ \rightarrow e^+e^-\mu^+\mu^-$  [9].

In conclusion, we have demonstrated that, for an arbitrary amount of  $CP$  violation in the  $HVV$  coupling, the distribution (2) has the compact and transparent form (17). As compared with the observables  $A'$  and  $B'$ , presence of  $CP$  violation would be much more easily measured by using the energy-weighted observable  $C'$ . Finally, having established  $CP$  violation, the actual strength may be determined from measurements of the observables  $C$  and  $C'$ .

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### Figure captions

- Fig. 1. The amplitude functions  $\alpha(m)\rho^2$  and  $-\beta(m)\rho$  of (17) in the case  $H \rightarrow W^+W^- \rightarrow 4f$ , for a Higgs of mass  $m$ , and for  $\eta = 1, 10^{-1}$  and  $10^{-3}$ .
- Fig. 2. The amplitude functions  $\alpha(m)\rho^2$  and  $-\beta(m)\rho$  of (17) in the case  $H \rightarrow ZZ \rightarrow 4l$ , for a Higgs of mass  $m$  and for  $\eta = 1, 10^{-1}$  and  $10^{-3}$ .
- Fig. 3. The angle  $\delta$  (in degrees) for a Higgs particle of mass  $m$ , for  $\eta = 1, 10^{-1}, 10^{-2}$ , and  $10^{-3}$ .
- Fig. 4. The ratio  $C'(m)/B'(m)$  for a Higgs of mass  $m$  in the cases of  $H \rightarrow W^+W^- \rightarrow 4f$  and  $H \rightarrow ZZ \rightarrow 4l$ , for  $\eta = 10^{-3}, 10^{-1}$ , and  $1$ .
- Fig. 5. The amplitude function  $-\kappa(m)/\rho$  in the cases  $H \rightarrow W^+W^- \rightarrow 4f$  and  $H \rightarrow ZZ \rightarrow 4l$ , for a Higgs of mass  $m$ , and for  $\eta = 1, 10^{-1}$ , and  $10^{-3}$ .

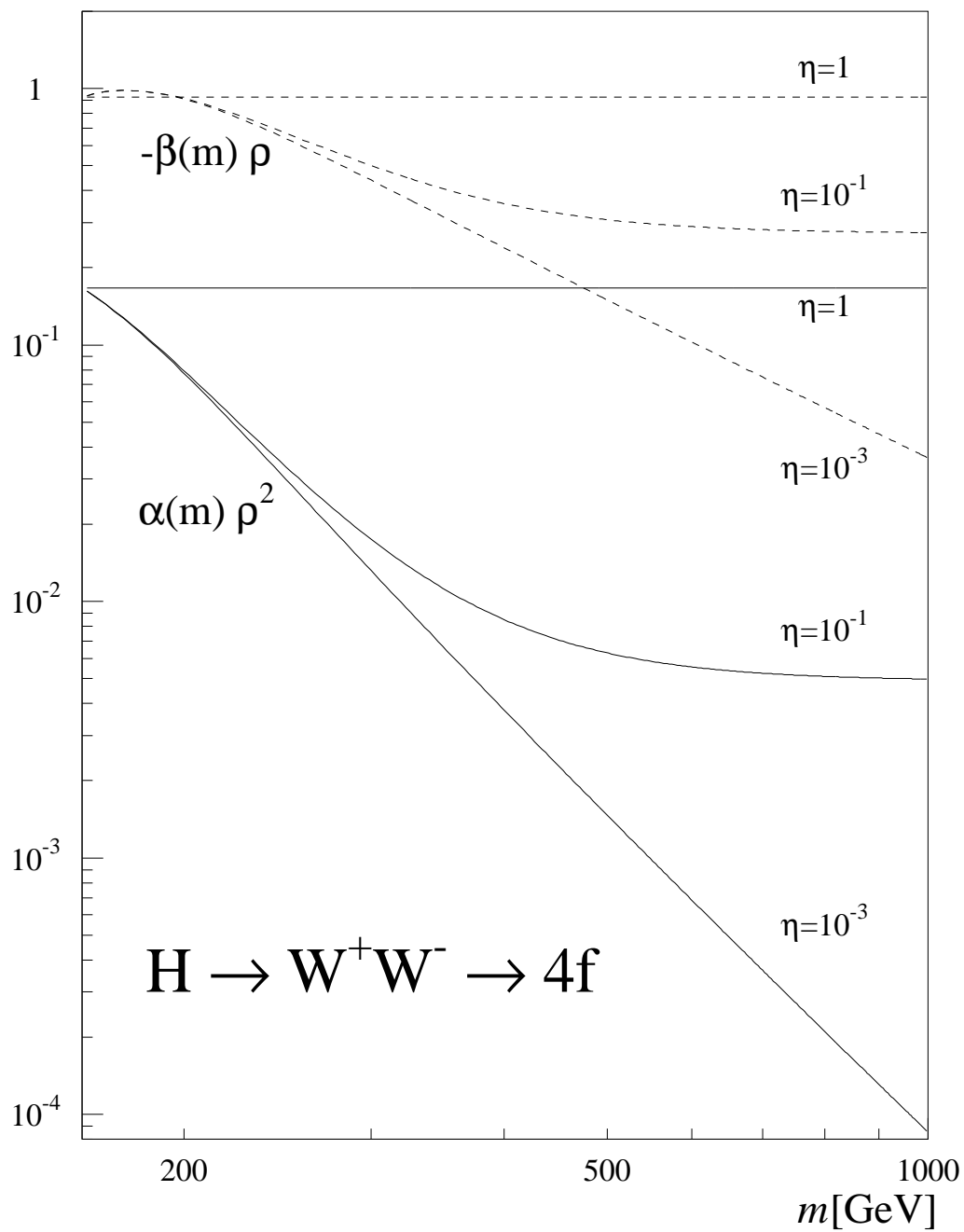


Figure 1

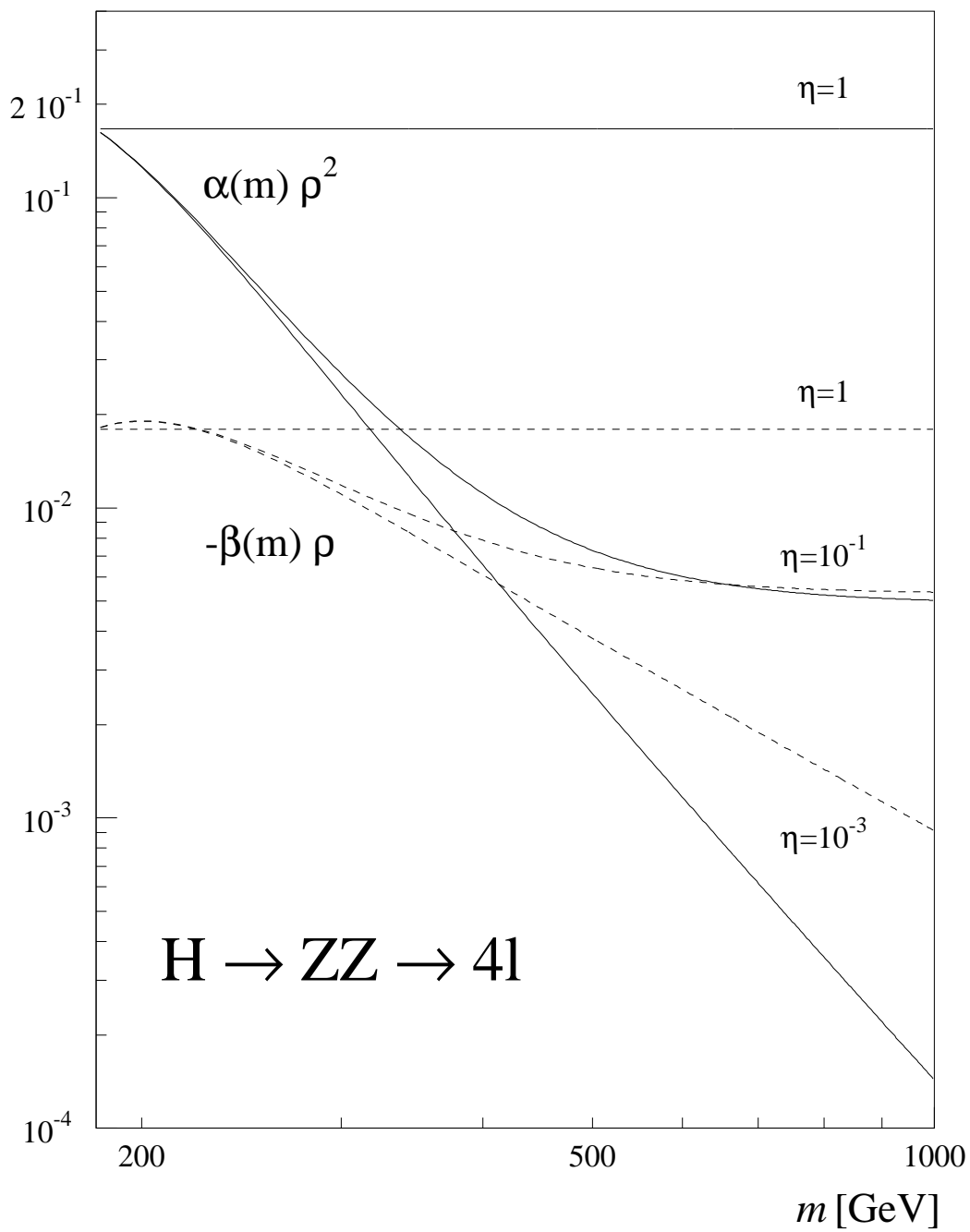


Figure 2

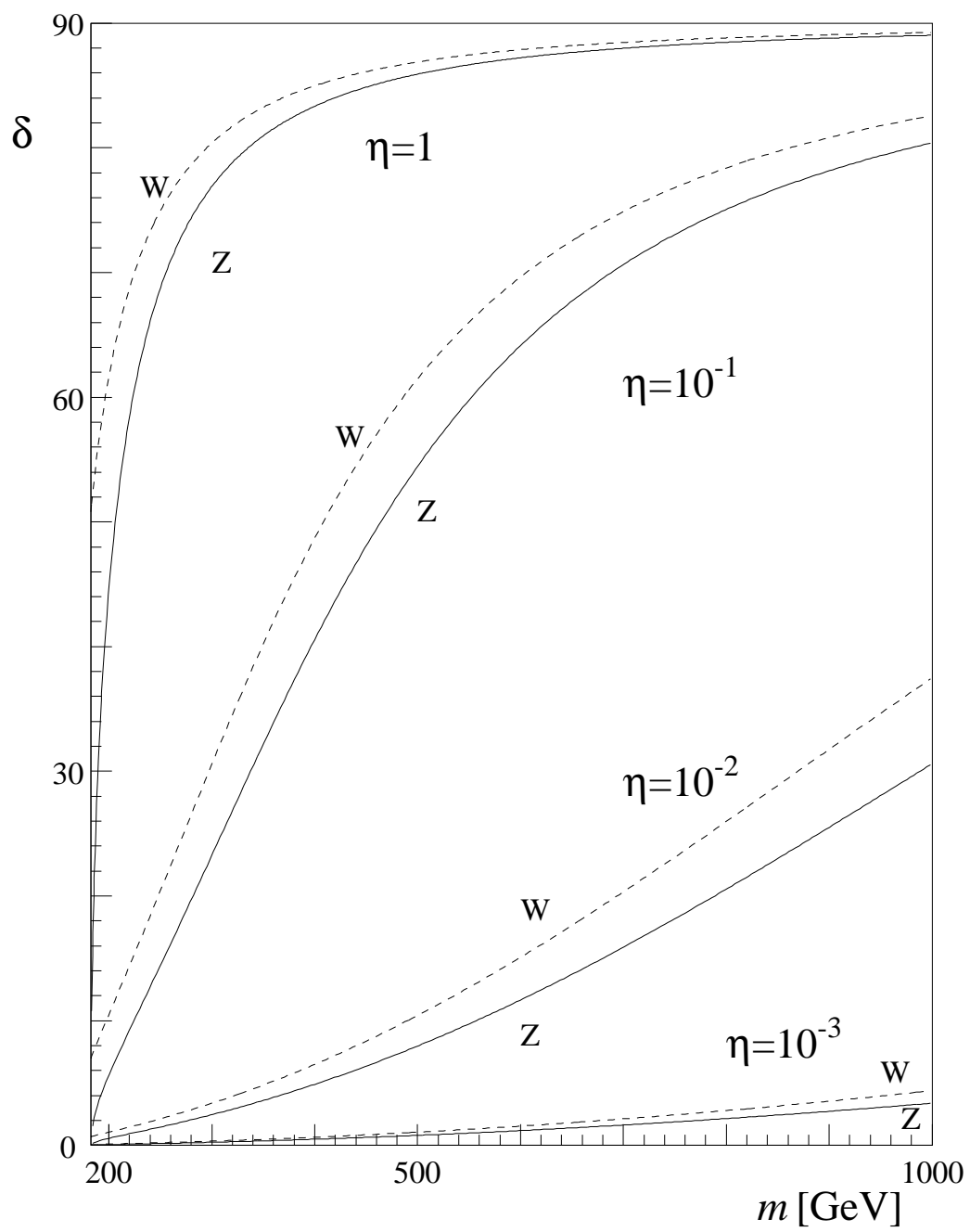


Figure 3

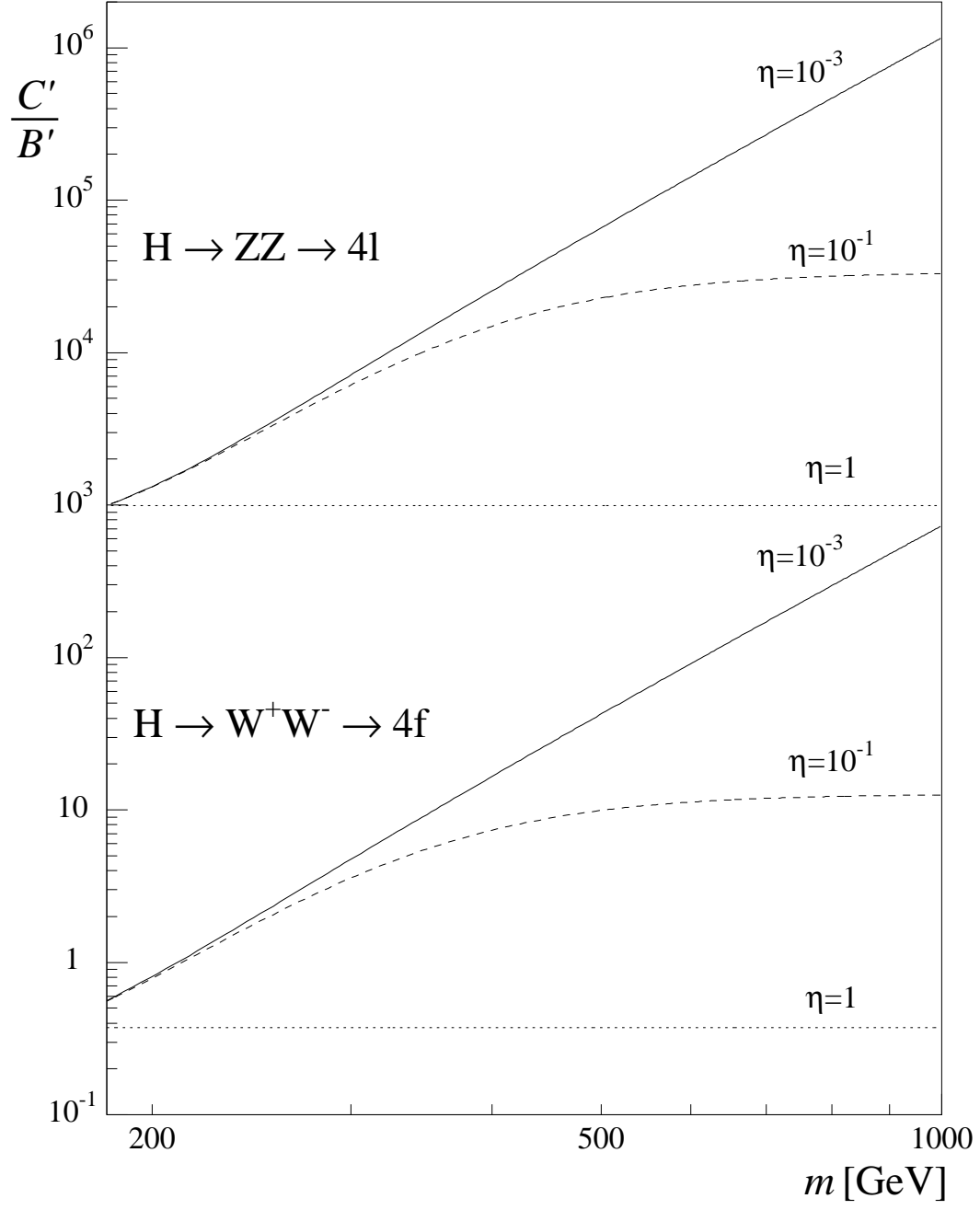


Figure 4

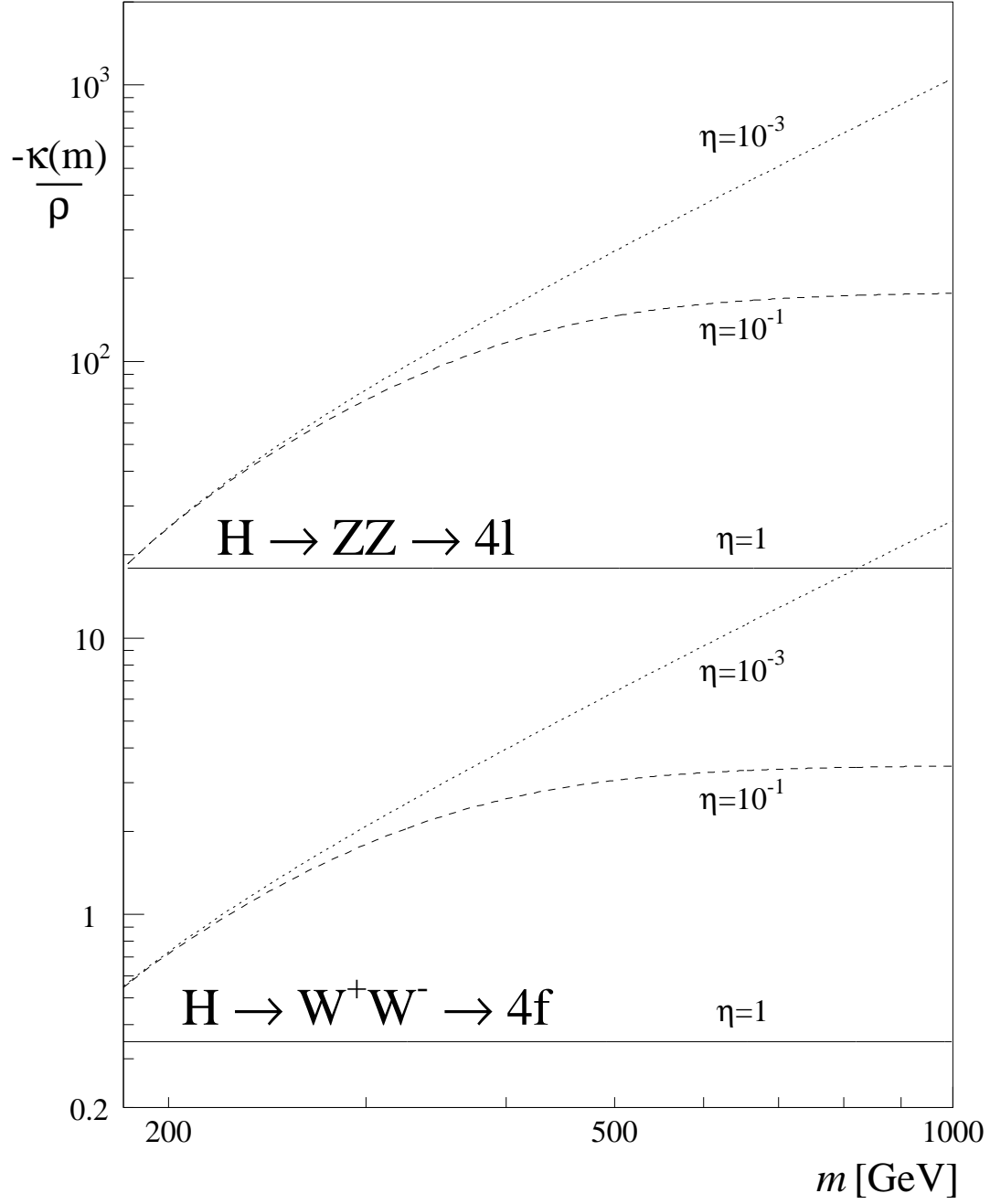


Figure 5